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Whistlers and Negative Ions

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Using an appropriate model of the ionosphere, we find the travel time for proton whistlers to go from their source to an observer at a satellite. The results differ from earlier ones. The physical parameters obtained through whistler observations agree with these results. Damping and attenuation of whistlers are related to the ionospheric parameters.

1. INTRODUCTION

In the last few decades, the study of the propagation of ion whistlers has greatly increased and has been employed as a diagnostic technique to estimate ionospheric parameters by many workers, especially Gurnett and his group (Gurnett, 1965; Gurnett and Shawhan, 1966; Gurnett and Brice, 1966), who were the first to make a rigorous and extensive study of whistlers in the ionosphere. Subsequent observations of whistler features intensified further the mathematical theory of waves in ionospheric plasmas (Lucas and Brice, 1971; Singh *et al.*, 1976; Das and Sur, 1986). Such studies are based on the measurement of the group travel time of a whistler from its source to an observer at a satellite and is given by the line integral

$$t(\omega)=\int_0^h \frac{dh}{V_g} \tag{1}$$

where V_g is the group velocity along the path of propagation. Gurnett and co-workers found that the right circularly polarized wave, when the frequency is nearly equal to the ion-cyclotron frequency, contributes negligibly to the group travel time as compared to that of the left circularly polarized wave. But the present model of the ionosphere shows that both circularly polarized waves contribute to the group velocity V_g, and thus the calculation of $t(\omega)$ should differ from the earlier mathematical treatment.

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These phenomena arise only when the model of the ionosphere consists of both kinds of ions. The existence of negative ions in the ionosphere (Branscomb, 1964; Von Goeler *et al.*, 1966; Whitten *et al.*, 1965; Das, 1975) leads to an improved treatment and implies that the interaction of the negative ions cannot be ignored. Das and Sur (1986) made an initial study of the group travel time, which the present paper makes more detailed. Finally, the damping and the attenuation of the whistlers are studied. As expected, the negative ions contribute significantly to the group travel time, and if an appropriate model is not used, there is a discrepancy in the results.

2. MATHEMATICAL FORMULATION

We consider the ionosphere to consist of multicomponent ions of both kinds and electrons. The α-type charged particles having mass m_α and density n_α move with velocity v_α. We assume that in the equilibrium state, the medium is pervaded by a uniform magnetic field $\mathbf{H}=(0, 0, H)$. The basic equations governing the ionosphere are

$$\frac{\partial n_\alpha}{\partial t}+\operatorname{div}(n_\alpha \mathbf{v}_\alpha)=0 \tag{2}$$

$$m_\alpha\left[\frac{\partial \mathbf{v}_\alpha}{\partial t}+(\mathbf{v}_\alpha\cdot\nabla)\mathbf{v}_\alpha\right]=q_\alpha\left[\mathbf{E}+\frac{\mathbf{v}_\alpha\times\mathbf{H}}{c}\right] \tag{3}$$

supplemented by Maxwell's equations

$$\nabla\times\mathbf{H}=\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}+4\pi\mathbf{J} \tag{4}$$

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t} \tag{5}$$

$$\nabla\cdot\mathbf{H}=0 \tag{6}$$

$$\nabla\cdot\mathbf{E}=4\pi\sum_\alpha n_\alpha q_\alpha \tag{7}$$

where $\alpha=i$ stands for positive ions, $\alpha=j$ for negative ions, and $\alpha=e$ for electrons, and $q_\alpha=e$ when $\alpha=i$ and $q_\alpha=-e$ when $\alpha=j$, e. All other symbols have their usual meanings as defined in Stix (1962).

Further, we assume a plane wave propagating in such a way that all the perturbed parameters are functionally varying as $\exp[i(k\cdot r-\omega t)]$. Following Stix (1962), the dispersion relation for the propagation at an angle θ is obtained as

$$An^4-Bn^2+C=0 \tag{8}$$

where

$$A = S\sin^2\theta + P\cos^2\theta$$

$$B = RL\sin^2\theta + PS(1+\cos^2\theta)$$

$$C = PRL$$

$$R, L = 1 - \sum_i \frac{\omega_{pi}^2}{\omega(\omega \pm \omega_{ci})} - \sum_j \frac{\omega_{pj}^2}{\omega(\omega \mp \omega_{cj})} - \frac{\omega_{pe}^2}{\omega(\omega \mp \omega_{ce})}$$

$$S = \tfrac{1}{2}(R+L)$$

$$D = \tfrac{1}{2}(R-L)$$

$$P = 1 - \sum_\alpha \frac{\omega_{p\alpha}}{\omega^2}, \qquad \text{where} \quad \omega_{p\alpha}^2 = \frac{4\pi e^2 n_0}{m_\alpha}$$

n_0 is the initial density, $\omega_{c\alpha} = |q_\alpha| H/cm_\alpha$, and $n = ck/\omega$ is the refractive index.

The satellite observations show that very few whistlers can be seen at the equator as compared to the occurrence rates at midlatitude. These features of whistlers have made it possible for researchers to develop a mathematical theory for whistlers at midlatitude. In such cases, the propagation angle is small and the dispersion relation (8), after some straightforward mathematical manipulation, reduces to

$$n^2 = \left(\frac{1+\chi}{2}\right) \frac{\omega_{pi}^2 \sec^2\theta}{\omega_{ci}(\omega_{ci} - \omega)} \tag{9}$$

where $\chi = n_{j0}/n_{i0}$ is the concentration ratio between negative ions and positive ions.

The corresponding group velocity is determined as

$$V_g = \left(\frac{2}{1+\chi}\right)^{1/2} \frac{c(\omega_{ci}(0))^{1/2}(\omega_{ci}(0) - \omega)^{3/2}}{\omega_{pi}(0)[\omega_{ci}(0) - \omega/2]} \tag{10}$$

which shows that the group velocity does not depend on the propagation angle of the wave. In order to evaluate the integral (1) with V_g in equation (10), we assume that the density variation in the ionosphere is negligibly small and that the magnetic field varies linearly as

$$\omega_{ci}(h) = \omega_{ci}(0) + h\omega'_{ci}(0) \tag{11}$$

where ω_{ci} is the cyclotron frequency at the satellite and $\omega'_{ci}(0)$ expresses the variation of the geomagnetic field. Following Gurnett (1965), straightforward mathematical manipulation gives the group travel time as

$$t(\omega) = \frac{1+\chi}{2} \frac{\omega_{pi}(0)[\omega_{ci}(0)]^{1/2}}{c\omega'_{ci}(0)[\Delta\omega(0)]^{1/2}} \tag{12}$$

Equation (12) is the required equation to determine the plasma parameters in the ionosphere. To determine the physical parameters, we plot the travel time $t(\omega)$ against $[\Delta\omega(0)]^{1/2}$ for several wave frequencies with an arbitrarily chosen cyclotron frequency. The plot is continued with the change of the cyclotron frequency until the plot gives a straight line. The cyclotron frequency for which a straight line is exhibited is the required ion-cyclotron frequency. The slope of the line indicates the density of the medium. Elaborate numerical estimations have been given by Das and Sur (1986), who concluded that the presence of the negative ions in the ionosphere should be considered or otherwise the method leads to a discrepancy in the results.

3. FURTHER USE OF GROUP TRAVEL TIME

We further extend the use of the group travel time to calculate the damping of the whistlers and so estimate the other physical parameters of the ionosphere. The satellite observations indicate that a low-frequency whistler in the ionosphere always propagates with a rising tone and approaches the ion-cyclotron frequency. But when the wave propagates with frequency almost equal to the ion-cyclotron frequency, the charged particles moving along the magnetic field observe a high-frequency wave. In such cases, most of the ions absorb energy from the field and consequently damping occurs in the whistler. To calculate the damping rate, we assume that the velocity distribution of the charged particles is Maxwellian. Following Stix (1966), the dispersion relations of circularly polarized waves propagating along the magnetic field are

$$L = 1 + \frac{\omega_{pi}^2}{\omega_{ci}(\omega_{ci} - \omega)} + \frac{2\sqrt{\pi}\,\omega_{pi}^2}{\omega|k|v_{ti}} \exp(-z_i^2) \tag{13}$$

$$R = 1 + \frac{\omega_{pj}^2}{\omega_{cj}(\omega_{cj} - \omega)} + \frac{2\sqrt{\pi}\,\omega_{pj}^2}{\omega|k|v_{tj}} \exp(-z_j^2) \tag{14}$$

where $z_\alpha = \Delta\omega/kv_{t\alpha}$, and $v_{t\alpha}$ is the thermal velocity of the α-type ionic species. We assume that the temperatures of the ions are the same, to simplify the calculations. For the present analysis, we consider a simplified dispersion relation in the form

$$n^2 = 1 + \left(\frac{1+\chi}{2}\right) \frac{\omega_{pi}^2}{\omega_{ci}(\omega_{ci} - \omega)} + \frac{1+\chi}{2} \frac{i\sqrt{\pi}\,\omega_{pi}^2}{\omega|k|v_{ti}} \exp(-z^2) \tag{15}$$

where $z_\alpha = z$.

The calculation of the damping in the whistler wave depends on the refractive index given by equation (15) as well as on the group travel time expression (12). We assume first that k is real, and the wave frequency ω is expressed as $\omega=\omega_r+i\omega_i$, with $|\omega_i|\ll|\omega_r|$. The substitution of ω in (15) and the separation of the imaginary part gives ω_i as

$$\omega_i \simeq -\left(\frac{2}{1+\chi}\right)^{1/2} \frac{c\sqrt{\pi}\,[\omega_{ci}(0)]^{1/2}[\Delta\omega(0)]^3}{2v_{ti}\omega_{pi}(0)} \exp[\bar{\eta}(0)] \tag{16}$$

where

$$\bar{\eta}(h) = -\frac{2}{1+\chi} \frac{c^2[\Delta\omega(h)]^3}{\omega_{ci}(h)\omega_{pi}^2(h)v_{ti}^2}$$

it is assumed that $\Delta\omega(0)$ is small. The damping rate ω_i with the use of the group travel time $t(\omega)$ reduces to the following form:

$$\omega_i \simeq A t^{-3} \exp(-Bt^{-6}) \tag{17}$$

together with

$$A = -\left(\frac{1+\chi}{2}\right)\frac{\sqrt{\pi}}{2v_{ti}}\left[\frac{\omega_{ci}(0)\omega_{pi}(0)}{c}\right]^2 \Big/ [\omega'_{ci}(0)]^3$$

$$B = \left(\frac{1+\chi}{2}\right)\left(\frac{\omega_{pi}(0)}{c}\right)^4\left(\frac{\omega_{ci}(0)}{v_{ti}}\right)^2 \Big/ [\omega'_{ci}(0)]^3$$

Again, the damping rate of the ion whistler wave is believed to be related (Lucas and Brice, 1971) to the velocity distribution function of the charged particles through the relation

$$\omega_i \simeq \frac{2c\pi^2}{\omega_{pi}(0)} \frac{[\Delta\omega(0)]^{5/2}}{[\omega_{ci}(0)]^{1/2}} F(v) \tag{18}$$

where $F(v)$ is the velocity distribution function of the charged particles, which, with the help of equation (12), is obtained in the form

$$F(v) = D \exp(-Bt^{-6}) \tag{19}$$

where

$$D = \left(\frac{2}{1+\chi}\right)^{1/2} \frac{1}{4\pi^{3/2}v_{ti}} \frac{\omega_{ci}(0)}{\Delta\omega(0)}$$

The expressions for the damping rate and the velocity distribution function given by equations (18) and (19), respectively, depend on the value of $\Delta\omega(0)$, which can be obtained from the group travel time given by (12). So, the knowledge of the group travel time for a whistler propagating from the source to an observer in the ionosphere gives easily the evaluation of the damping rate by using equation (17) and the velocity distribution function of the charged particles from expression (19). To show the qualitative variation on the damping rate, we consider a small percentage of negative ion concentration, $\chi = 0.1$, as compared to the positive ions. The variation against the group travel time is shown in Figure 1 for temperatures 600, 800, and 1000 K. Figure 1 shows the effects of the temperature as well as the travel time of the whistlers received by the satellite. The damping is negligible at the source of the whistler. But when the travel time $t(\omega)$ is more than 3 sec, the damping rises sharply even with a small increase of the travel time and effectively depends on the variation of the temperatures. Thus, Figure 1 shows that the presence of the negative ions has an overall effective contribution to the damping of the whistler wave. The numerical values of damping in the ionosphere are 14.2, 15.3, and 15.4 rad sec^{-1} for

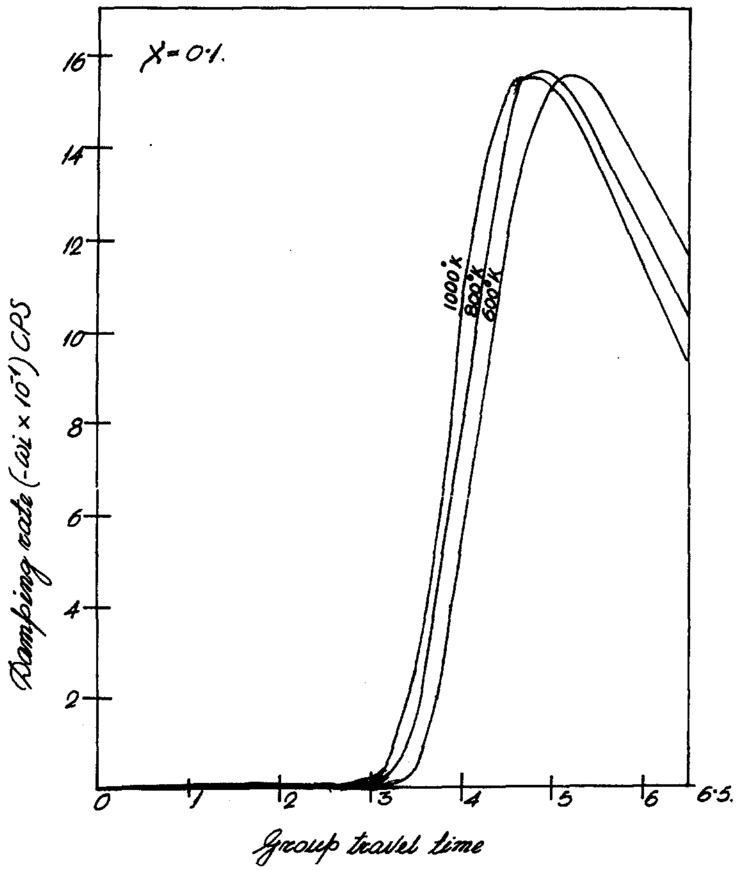

Fig. 1. Variation of temporal damping rate with group travel time.

temperatures 600, 800, and 1000 K, respectively, showing an increase in damping rate as compared to the results obtained in Gurnett's model. The maximum damping rate occurs near $t = 4.75$ sec and afterward the damping shows an opposite nature, decreasing with the group travel time.

Figure 2 shows the variation in the velocity distribution function $F(v)$ of the particles with the group travel time. The variation appears negligibly small near the source of the whistlers, while a sharp steepening occurs at about $t = 3$ sec and afterward. The negative-ion interaction with the whistler wave shows that the range of distribution function variation remains

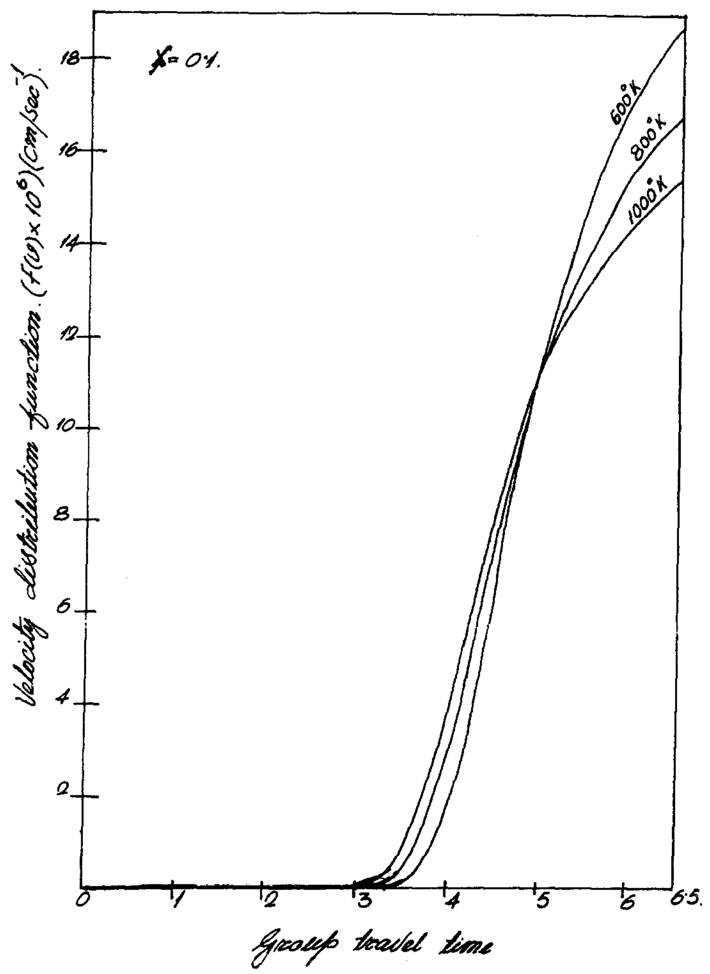

Fig. 2. Variation of velocity distribution function with group travel time.

unaffected near the source of the whistler and is widened as compared to the results in the simpler case. The variation of the distribution function gives the resonating ionic number density and is calculated by multiplying the velocity distribution function with the number density of the ion.

Now we consider the case when the wave frequency ω is real and the propagation vector k is expressed as $k = k_r + ik_i$, with $|k_i| \ll |k_r|$. Again, the separation of the imaginary part of the propagation vector from equation (15) gives the cyclotron damping as

$$k_i = \frac{\sqrt{\pi}\,\Delta\omega(0)}{2v_{ti}} \exp[\bar{\eta}(0)] \tag{20}$$

where $\bar{\eta}(0)$ is defined in equation (16) and similar assumptions are also made here. The corresponding total attenuation of the wave is calculated by integrating k_i over the propagation path using the same assumption and procedure as given by Gurnett and Brice (1966):

$$\begin{aligned}\beta &= 2\int_0^h k_i\,dh \\ &\simeq \frac{1+\chi}{2}\,\frac{\sqrt{\pi}}{3c^2}\,\frac{\omega_{pi}^2(0)\omega_{ci}(0)v_{ti}}{\omega'_{ci}(0)\,\Delta\omega(0)} \exp[\bar{\eta}(0)]\end{aligned} \tag{21}$$

The value of $\Delta\omega(0)$ can be substituted from the group travel time (12) and a calculation gives equation (21) in the form

$$\beta = A't^2 \exp(-Bt^{-6}) \tag{22}$$

where $A' = \sqrt{\pi}\,\omega'_{ci}(0)v_{ti}/3$ and B is as in (17).

The relation of the attenuation of the whistler to the amplitude of the magnetic field gives the magnetic field as

$$\begin{aligned}B_1 &\propto e^{-\beta} \\ &\propto t^{-1} \exp[-A't^2 \exp(-Be^{-6})]\end{aligned} \tag{23}$$

A similar expression has been derived by Gurnett and Brice (1966), with the main differences in the derivation of the group travel time. The change of the group travel time arises due to the presence of the negative ions in the ionosphere, leading to an overall significant change in the mathematical treatment of the estimation of the physical parameters. We plot the variation of the attenuation against the group travel time in Figure 3. Figure 3 shows a delay of the attenuation; it cannot be seen at the source of the whistler. After a certain time, approximately 3 sec, the attenuation rises as the whistler wave progresses. Figure 4 shows the variation of the amplitude of the magnetic field, and a comparison of the results with that of Gurnett and

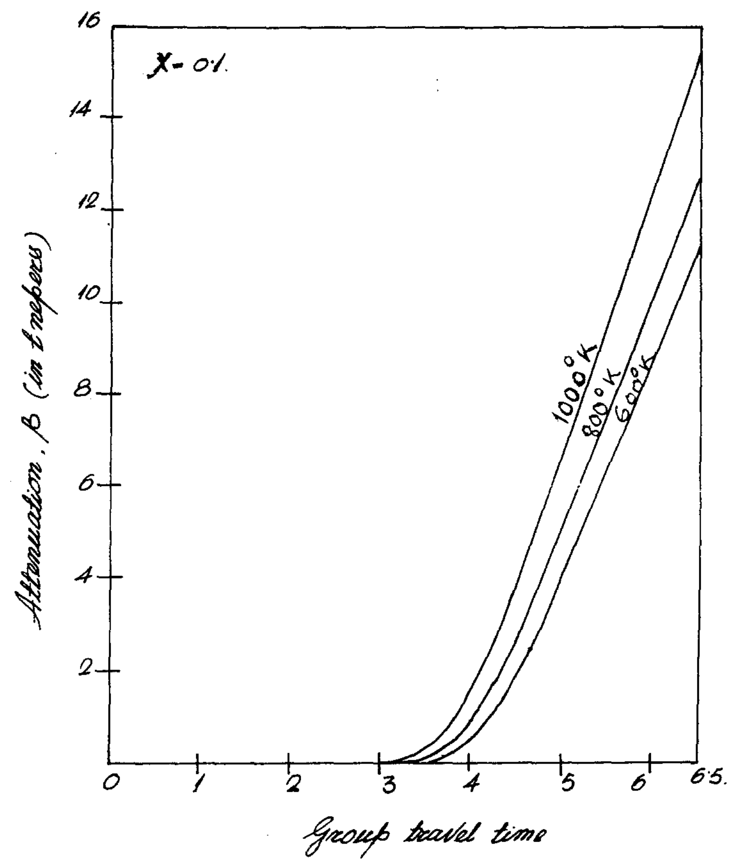

Fig. 3. Variation of attenuation β with respect to group travel time.

Brice (1966). The amplitude variation is sensitively influenced by the presence of the negative ions. For a typical value of $t(\omega) = 3$ sec, Gurnett and Brice (1966) calculated B_1 as nearly equal to 2.5×10^{-5}, while in our model B_1 increases to 0.3, for the temperature 600 K.

4. CONCLUSIONS

We have considered an appropriate model of the ionosphere consisting of multiple ions with different charges, with a view to showing the effects of negatively charged particles on the whistlers observed in the ionosphere. The characteristic variation of the damping and the attenuation of whistler propagation with group travel time are also shown. As expected, the calculation shows that at the equator the negative ions make a considerable

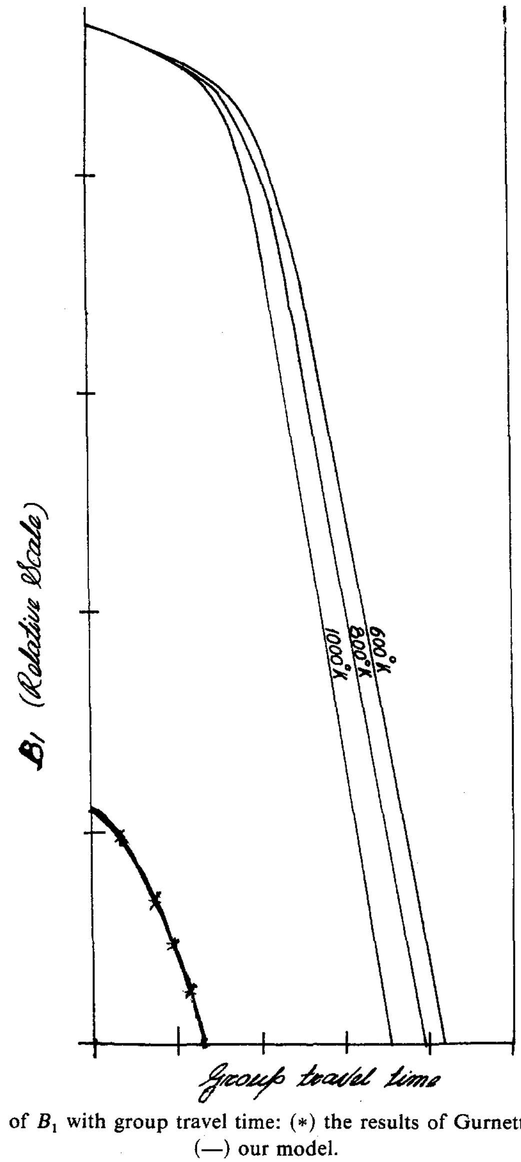

Fig. 4. Variation of B_1 with group travel time: (*) the results of Gurnett and Brice (1966); (—) our model.

contribution to the whistler wave. The presence of the negative ions delays the onset of damping and, when the damping is observed, leads to a sharp rise with increasing group travel time. Similar characteristics are also seen in the case of the amplitude of the magnetic field variation.

REFERENCES

Branscomb, L. M. (1964). *Annals of Geophysics* **20**, 88.
Das, G. C. (1975). *IEEE Transactions Plasma Science*, **3**, 168.
Das, G. C. (1979). *Astrophysics and Space Science*, **66**, 505.
Das, G. C., and Sur, A. K. (1986). *Astrophysics and Space Science*, **127**, 249.
Gurnett, D. A. (1965). Ph.D. Dissertation, University of Iowa.
Gurnett, D. A. and Brice, N. M. (1966). *Journal of Geophysical Research*, **71**, 3639.
Gurnett, D. A. and Shawhan, S. D. (1966). *Journal of Geophysical Research*, **71**, 741.
Lucas, C., and Brice, N. M. (1971). *Journal of Geophysical Research*, **76**, 92.
Singh, S. N., Tiwari, S. N., and Tolpadi, S. K. (1976). *Journal of Geophysical Research*, **81**, 1327.
Stix, T. H. (1962). *The Theory of Plasma Waves*, McGraw-Hill, New York.
Von Goeler, S., Ohe, T., and D'Angelo, N. (1966). *Journal of Applied Physics*, **37**, 2519.
Whitten, R. C., Poppoff, I. G., Edmonds, R. S., and Berning, W. W. (1965). *Journal of Geophysical Research*, **70**, 1737.